Trigonometrie

Valori, identități, formule de transformare, ecuații · liceu, pregătire pentru Bac

Valori ale funcțiilor trigonometrice

Unghi	sin	cos	tg	ctg
0° (0)	0	1	0	—
30° $(\frac{\pi}{6})$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45° $(\frac{\pi}{4})$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60° $(\frac{\pi}{3})$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90° $(\frac{\pi}{2})$	1	0	—	0
180° (π)	0	-1	0	—
270° $(\frac{3\pi}{2})$	-1	0	—	0
360° (2π)	0	1	0	—

Radiani: $\alpha_{\text{rad}} = \alpha^\circ \cdot \dfrac{\pi}{180}$. Lungimea arcului: $\ell = r\alpha$ (α în radiani).

Relații fundamentale

Formula	Condiții
$\sin^2 x + \cos^2 x = 1$	$x \in \mathbb{R}$
$\operatorname{tg} x = \dfrac{\sin x}{\cos x}$, $\operatorname{ctg} x = \dfrac{\cos x}{\sin x}$	$\cos x \neq 0$; respectiv $\sin x \neq 0$
$\operatorname{tg} x \cdot \operatorname{ctg} x = 1$	$x \neq \frac{k\pi}{2}$
$1 + \operatorname{tg}^2 x = \dfrac{1}{\cos^2 x}$	$x \neq \frac{\pi}{2} + k\pi$
$1 + \operatorname{ctg}^2 x = \dfrac{1}{\sin^2 x}$	$x \neq k\pi$

Paritate, periodicitate, reducere la primul cadran

Formula	Formula
$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$
$\operatorname{tg}(-x) = -\operatorname{tg} x$	$\operatorname{ctg}(-x) = -\operatorname{ctg} x$
$\sin(\pi - x) = \sin x$	$\cos(\pi - x) = -\cos x$
$\sin(\pi + x) = -\sin x$	$\cos(\pi + x) = -\cos x$
$\sin(2\pi - x) = -\sin x$	$\cos(2\pi - x) = \cos x$
$\sin\left(\frac{\pi}{2} - x\right) = \cos x$	$\cos\left(\frac{\pi}{2} - x\right) = \sin x$
$\operatorname{tg}(\pi - x) = -\operatorname{tg} x$	$\operatorname{tg}(x + \pi) = \operatorname{tg} x$
$\sin(x + 2k\pi) = \sin x$	$\cos(x + 2k\pi) = \cos x$, $k \in \mathbb{Z}$

Formule de adunare

Formula	Condiții
$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$	
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$	
$\operatorname{tg}(a \pm b) = \dfrac{\operatorname{tg} a \pm \operatorname{tg} b}{1 \mp \operatorname{tg} a \operatorname{tg} b}$	$a, b, a \pm b \neq \frac{\pi}{2} + k\pi$; $1 \mp \operatorname{tg} a \operatorname{tg} b \neq 0$
$\operatorname{ctg}(a + b) = \dfrac{\operatorname{ctg} a \operatorname{ctg} b - 1}{\operatorname{ctg} a + \operatorname{ctg} b}$	$a, b, a + b \neq k\pi$; $\operatorname{ctg} a + \operatorname{ctg} b \neq 0$

Unghi dublu, triplu, pe jumătate

Formula	Condiții
$\sin 2a = 2\sin a \cos a = \dfrac{2\operatorname{tg} a}{1 + \operatorname{tg}^2 a}$	$a \neq \frac{\pi}{2} + k\pi$ (pentru forma cu tg)
$\cos 2a = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a = \dfrac{1 - \operatorname{tg}^2 a}{1 + \operatorname{tg}^2 a}$	$a \neq \frac{\pi}{2} + k\pi$ (pentru forma cu tg)
$\operatorname{tg} 2a = \dfrac{2\operatorname{tg} a}{1 - \operatorname{tg}^2 a}$	$a \neq \frac{\pi}{2} + k\pi$, $a \neq \frac{\pi}{4} + \frac{k\pi}{2}$
$\sin 3a = 3\sin a - 4\sin^3 a$, $\cos 3a = 4\cos^3 a - 3\cos a$	
$\sin^2 \dfrac{a}{2} = \dfrac{1 - \cos a}{2}$, $\cos^2 \dfrac{a}{2} = \dfrac{1 + \cos a}{2}$	
$\operatorname{tg} \dfrac{a}{2} = \dfrac{\sin a}{1 + \cos a} = \dfrac{1 - \cos a}{\sin a}$	$a \neq \pi + 2k\pi$; (a doua formă) $a \neq k\pi$

Transformări de sumă în produs și invers

Sumă în produs	Produs în sumă
$\sin a + \sin b = 2\sin \dfrac{a+b}{2} \cos \dfrac{a-b}{2}$	$\sin a \cos b = \dfrac{\sin(a+b) + \sin(a-b)}{2}$
$\sin a - \sin b = 2\sin \dfrac{a-b}{2} \cos \dfrac{a+b}{2}$	$\sin a \sin b = \dfrac{\cos(a-b) - \cos(a+b)}{2}$
$\cos a + \cos b = 2\cos \dfrac{a+b}{2} \cos \dfrac{a-b}{2}$	$\cos a \cos b = \dfrac{\cos(a-b) + \cos(a+b)}{2}$
$\cos a - \cos b = -2\sin \dfrac{a+b}{2} \sin \dfrac{a-b}{2}$	

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Funcții trigonometrice și inverse

Funcția	Domeniu	Imagine	Perioada	Paritate
$\sin$	$\mathbb{R}$	$[-1,1]$	2π	impară
$\cos$	$\mathbb{R}$	$[-1,1]$	2π	pară
tg	$\mathbb{R}\setminus\left\{\frac{\pi}{2}+k\pi\right\}$	$\mathbb{R}$	π	impară
ctg	$\mathbb{R}\setminus\{k\pi\}$	$\mathbb{R}$	π	impară
$\arcsin$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	—	impară
$\arccos$	$[-1,1]$	$[0,\pi]$	—	$\arccos(-x)=\pi-\arccos x$
arctg	$\mathbb{R}$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$	—	impară
arcctg	$\mathbb{R}$	$(0,\pi)$	—	$\operatorname{arcctg}(-x)=\pi-\operatorname{arcctg} x$

Formula	Condiții
$\arcsin x+\arccos x=\frac{\pi}{2}$	$x\in[-1,1]$
$\operatorname{arctg} x+\operatorname{arcctg} x=\frac{\pi}{2}$	$x\in\mathbb{R}$
$\arcsin(-x)=-\arcsin x$, $\operatorname{arctg}(-x)=-\operatorname{arctg} x$	
$\arccos(-x)=\pi-\arccos x$	$x\in[-1,1]$
$\sin(\arcsin x)=x$; $\arcsin(\sin x)=x$	$x\in[-1,1]$; respectiv $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\operatorname{tg}(\operatorname{arctg} x)=x$; $\operatorname{arctg}(\operatorname{tg} x)=x$	$x\in\mathbb{R}$; respectiv $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Ecuații trigonometrice fundamentale

Ecuația	Soluțiile	Condiții
$\sin x=a$	$x=(-1)^k\arcsin a+k\pi$	$\lvert a\rvert\le 1$, $k\in\mathbb{Z}$
$\cos x=a$	$x=\pm\arccos a+2k\pi$	$\lvert a\rvert\le 1$, $k\in\mathbb{Z}$
$\operatorname{tg} x=a$	$x=\operatorname{arctg} a+k\pi$	$a\in\mathbb{R}$, $k\in\mathbb{Z}$
$\operatorname{ctg} x=a$	$x=\operatorname{arcctg} a+k\pi$	$a\in\mathbb{R}$, $k\in\mathbb{Z}$
$\sin x=\sin y$	$x=y+2k\pi$ sau $x=\pi-y+2k\pi$	$k\in\mathbb{Z}$
$\cos x=\cos y$	$x=\pm y+2k\pi$	$k\in\mathbb{Z}$
$\operatorname{tg} x=\operatorname{tg} y$	$x=y+k\pi$	$k\in\mathbb{Z}$

Triunghiul oarecare

Formula	Condiții
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$ (teorema sinusurilor)	R: raza cercului circumscris
$a^2=b^2+c^2-2bc\cos A$ (teorema cosinusului)	
$\mathcal{A}=\frac{1}{2}bc\sin A=\frac{1}{2}ab\sin C=\frac{1}{2}ac\sin B$	
$\operatorname{tg}\frac{A}{2}=\frac{r}{p-a}$	$p=\frac{a+b+c}{2}$, r: raza cercului înscris

Substituția universală

Substituția $t=\operatorname{tg}\frac{x}{2}$	Condiții
$\sin x=\frac{2t}{1+t^2}$	$x\neq\pi+2k\pi$
$\cos x=\frac{1-t^2}{1+t^2}$	$x\neq\pi+2k\pi$
$\operatorname{tg} x=\frac{2t}{1-t^2}$	$x\neq\pi+2k\pi$, $t\neq\pm1$